Math 55 Quiz 8 DIS 105

Name: _____ 18 Apr 2022

1. Find the solution to the recurrence relation $a_n = 6a_{n-1} - 8a_{n-2}$ with initial conditions $a_0 = 4, a_1 = 10$. [5 points]

   The characteristic polynomial to the recurrence relation is $x^2 = 6x - 8$, which has solutions $x = 2, 4$. Hence the general solution is $a_n = C_1 \cdot 2^n + C_2 \cdot 4^n$.
   From the initial conditions, we know that

$$C_1 + C_2 = 4$$
$$2C_1 + 4C_2 = 10$$

   Hence $C_1 = 3, C_2 = 1$, and the solution is $a_n = 3 \cdot 2^n + 4^n$.

2. Encrypt the message `16` using the RSA cryptosystem with key $(5 \cdot 7, 5)$. (Your answer should be a number between 0 and 34.) [5 points]

   To encrypt `16`, we have to compute $16^5 \pmod{5 \cdot 7 = 35}$. This can be done using fast modular exponentiation:
   $16^2 = 256 \equiv 11 \pmod{35}$
   $16^4 \equiv 11^2 = 121 \equiv 16 \pmod{35}$
   So $16^5 = 16 \cdot 16^4 \equiv 16 \cdot 16 = 256 \equiv 11 \pmod{35}$.